MATHCOUNTS®

2012
■ State Competition ■
Sprint Round
Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

1. _____ minutes

A bucket is filled with 10 gallons of water. A hole is created in the bucket so that it loses 2 gallons of water every 10 minutes. After how many minutes will the bucket be completely empty?

2. _____

The sum of the digits of a positive, two-digit integer is 9. The positive difference between the integer created when the two digits are reversed and the original integer is 27. What is the product of these two digits?

3. _____ diagonals

How many diagonals does a convex octagon have?

4. _____

Jean is twice as likely to make a free throw as she is to miss it. What is the probability that she will miss three free throws in a row? Express your answer as a common fraction.

5. _____ edges

A pyramid has 6 vertices and 6 faces. How many edges does it have?

6. _____

The product of the integers from 1 through 7 is equal to $2^j \cdot 3^k \cdot 5 \cdot 7$. What is the value of $j - k$?

7. _____

A standard six-sided die was rolled repeatedly. The frequency table below shows how many times each number was rolled. What is the mean of the 20 numbers rolled? Express your answer as a decimal to the nearest hundredth.

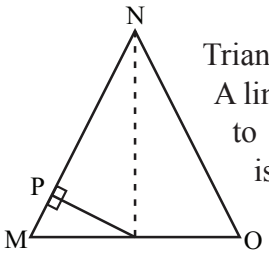
Number	1	2	3	4	5	6
Times rolled	4	4	3	4	2	3

8. _____ units^2 The coordinates of the vertices of a trapezoid are (1,7), (1,11), (8,4) and (4,4). What is the area of the trapezoid?

9. _____ seconds Malika ran 3 miles. She ran the first mile in 6 minutes, 45 seconds. Her time to complete each mile was $\frac{1}{9}$ longer than her time to complete the previous mile. In seconds, how long did it take Malika to run all 3 miles?

10. _____ Four consecutive integers are substituted in every possible way for distinct values a, b, c and d. What is the positive difference between the smallest and largest possible values of $(ab + cd)$?

11. _____ cm



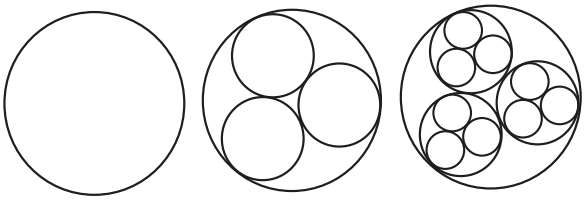
Triangle MNO is an isosceles triangle with $\overline{MN} = NO = 25$ cm. A line segment, drawn from the midpoint of $\overline{MO}$ perpendicular to $\overline{MN}$, intersects $\overline{MN}$ at point P with NP:PM = 4:1. What is the length of the altitude drawn from point N to $\overline{MO}$? Express your answer in simplest radical form.

12. _____ Let $\{a, b, c, d\}$ be a set of numbers chosen from the first nine positive integers. If you add every possible pair of these four numbers you get these sums: 7, 9, 10, 12, 13 and 15. What is the smallest possible product of these four numbers?

13. _____ In a sequence of positive integers, every term after the first two terms is the sum of the two previous terms in the sequence. If the fifth term is 2012, what is the maximum possible value of the first term?

14. _____ circles

The figure shows the first three stages of a fractal. If the pattern continues, how many circles will Stage 5 of the fractal contain?

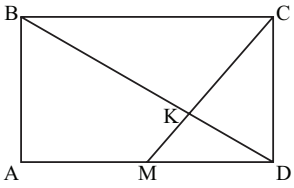


Stage 1 Stage 2 Stage 3

15. _____ pairs

Products are found by multiplying three different numbers from the set {1, 2, 3, 4, 5}. Among those products, how many pairs of relatively prime numbers are there?

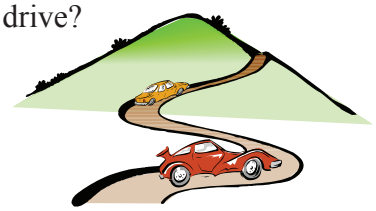
16. _____ units



In rectangle ABCD, AB = 6 units, the measure of ∠DBC is 30°, M is the midpoint of segment AD and segments CM and BD intersect at point K. What is the length of segment MK? Express your answer is simplest radical form.

17. _____ hours

Jack and Jill drove in separate cars to their favorite hill, leaving from the same place at the same time. Jill drove 20% faster than Jack and arrived half an hour earlier. How many hours did Jack drive?



18. _____

What is the largest five-digit integer such that the product of the digits is 2,520?

19. _____ unit cubes

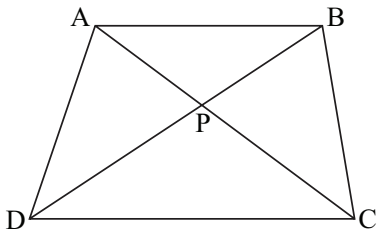
A rectangular prism is composed of unit cubes. The outside faces of the prism are painted blue and the seven unit cubes in the interior remain entirely unpainted. How many unit cubes have exactly one painted face?

20. _____ If $f(x) = x^2 + 5$ and $g(x) = 2(f(x))$, what is the greatest possible value of $f(x + 1)$ when $g(x) = 108$?

21. _____ cm A right triangle has sides with lengths 8 cm, 15 cm and 17 cm. A circle is inscribed in the triangle. In centimeters, what is the radius of the circle?

22. _____ Given that $x \neq 0$, what quantity can be added to $\frac{x+1}{x}$ or multiplied by $\frac{x+1}{x}$ to give the same result? Express your answer in terms of x.

23. _____ units^2 In trapezoid ABCD segments AB and CD are parallel. Point P is the intersection of diagonals AC and BD. The area of ΔPAB is 16 square units, and the area of ΔPCD is 25 square units. What is the area of trapezoid ABCD?

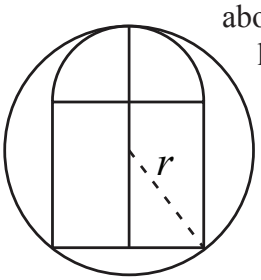


24. _____ Two positive numbers have the property that the sum of their squares is 20 and the sum of their reciprocals is 2. What is their product? Express your answer as a common fraction.

25. _____ A triangle has angles measuring 15°, 45° and 120°. The side opposite the 45° angle is 20 units in length, and the area of the triangle can be expressed in square units as $(m - n\sqrt{q})$, where q is a prime number. What is the value of the sum $m + n + q$?

26. _____ units

A silo-shaped plane figure is formed by positioning a semicircle above a square. The diameter of the semicircle is 2 units long and coincides with the top of the square. What is the radius, r, of the smallest circle that contains this figure? Express your answer as a common fraction.



27. _____ ways

In how many ways can six different gifts be given to five different children with each child receiving at least one gift and each gift being given to exactly one child?

28. _____

If the cost of a dozen eggs is reduced by x cents, a buyer will pay one cent less for $x + 1$ eggs than if the cost of a dozen eggs is increased by x cents. What is the value of x?

29. _____ subsets

For how many two-element subsets $\{a, b\}$ of the set $\{1, 2, 3, \ldots, 36\}$ is the product ab a perfect square?

30. _____

In rectangle ABCD, shown here, point M is the midpoint of side BC, and point N lies on $\overline{CD}$ such that DN:NC = 1:4. Segment BN intersects $\overline{AM}$ and $\overline{AC}$ at points R and S, respectively. If NS:SR:RB = x:y:z, where x, y and z are positive integers, what is the minimum possible value of $x + y + z$?