MATHCOUNTS®

2013
■ State Competition ■
Sprint Round
Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

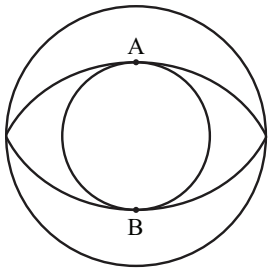
1. _____ If the sum of two numbers is 4 and their difference is 2, what is their product?

2. _____ miles Mary and Ann live at opposite ends of the same road. They plan to leave home at the same time and ride their bikes to meet somewhere between the two houses. At 11:00 a.m. Mary has traveled half of the distance between their houses. Ann is riding more slowly and has covered only $\frac{3}{8}$ of the distance between the houses. They are still one mile apart. How many miles apart are their houses?

3. _____ A bag contains 8 blue marbles, 4 red marbles and 3 green marbles. In a single draw, what is the probability of not drawing a green marble? Express your answer as a common fraction.

4. _____ The arithmetic mean of 11 numbers is 78. If 1 is subtracted from the first, 2 is subtracted from the second, 3 is subtracted from the third, and so forth, until 11 is subtracted from the eleventh, what is the arithmetic mean of the 11 resulting numbers?

5. _____ in^2 An optometrist has this logo on his storefront. The center circle has area 36π in^2, and it is tangent to each crescent at its widest point (A and B). The shortest distance from A to the outer circle is $\frac{1}{3}$ the diameter of the smaller circle. What is the area of the larger circle? Express your answer in terms of π.

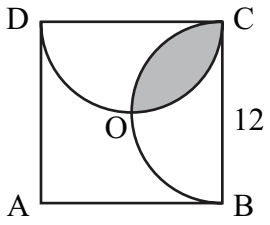


6. $\$$ _____ Excluding sales tax, how much will Doris save when she buys a DVD originally priced at \$12.00 and now on sale for 20% off?

7. _____ What is the value of $\dfrac{444^2 - 111^2}{444 - 111}$?

8. _____ The product of the digits of positive integer n is 20, and the sum of the digits is 13. What is the smallest possible value of n?

9. _____ cm^2 Quadrilateral ABCD is a square with BC = 12 cm. $\overset{\frown}{BOC}$ and $\overset{\frown}{DOC}$ are semicircles. In terms of π, what is the area of the shaded region?

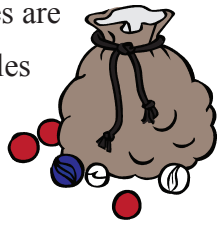


10. _____ Real numbers a and b satisfy the equation $\dfrac{2a-4}{5} + \dfrac{3a+1}{5} = b$. What is the value of $a - b$? Express your answer as a common fraction.

11. _____ If the point (x, x) is equidistant from $(-2, 5)$ and $(3, -2)$, what is the value of x?

12. _____ marbles

In a bag of marbles, $\frac{2}{5}$ of the marbles are red, $\frac{3}{10}$ of the marbles are white and $\frac{1}{10}$ of the marbles are blue. If the remaining 10 marbles are green, how many marbles are in the bag?



13. _____

If t is 40% greater than p, and p is 40% less than 600, what is the value of $t - p$?

14. _____ ways

How many ways can all six numbers in the set {4, 3, 2, 12, 1, 6} be ordered so that a comes before b whenever a is a divisor of b?

15. _____

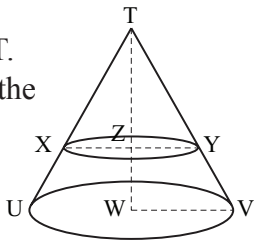
What is the units digit of the product $7^{23} \times 8^{105} \times 3^{18}$?

16. _____

If $4(a - 3) - 2(b + 5) = 14$ and $5b - a = 0$, what is the value of $a + b$?

17. _____ m^3

The two cones shown have parallel bases and common apex T. TW = 32 m, WV = 8 m and ZY = 5 m. What is the volume of the frustum with circle W and circle Z as its bases? Express your answer in terms of π.

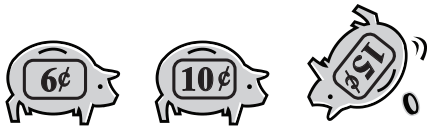


18. _____

A coin is flipped until it has either landed heads two times or tails two times, not necessarily in a row. If the first flip lands heads, what is the probability that a second head occurs before two tails? Express your answer as a common fraction.

19. _____ The product of two consecutive integers is five more than their sum. What is the smallest possible sum of two such consecutive integers?

20. _____ cents Four nickels, one penny and one dime were divided among three piggy banks so that each bank received two coins. Labels indicating the amount in each bank were made (6 cents, 10 cents and 15 cents), but when the labels were put on the banks, no bank had the correct label attached. Soraya shook the piggy bank labeled as 15 cents, and out fell a penny. What was the actual combined value of the two coins contained in the piggy bank that was labeled 6 cents?



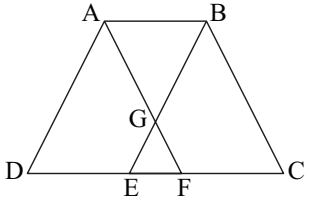
21. _____ Suppose the 9 × 9 multiplication grid, shown here, were filled in completely. What would be the sum of the 81 products?

×	1	2	3	4	5	6	7	8	9
1									
2							14		
3									
4									
5									
6		12							
7									
8					40				
9									

22. _____ "words" In some languages, every consonant must be followed by a vowel. How many seven-letter "words" can be made from the Hawaiian word MAKAALA if each consonant must be followed by a vowel?

23. _____ If $f(x) = 3x^2$, what is the x-coordinate of the point of intersection of the graphs of $y = f(x)$ and $y = f(x - 4)$?

24. _____ In isosceles trapezoid ABCD, shown here, AB = 4 units and CD = 10 units. Points E and F are on $\overline{CD}$ with $\overline{BE}$ parallel to $\overline{AD}$ and $\overline{AF}$ parallel to $\overline{BC}$. $\overline{AF}$ and $\overline{BE}$ intersect at point G. What is the ratio of the area of triangle EFG to the area of trapezoid ABCD? Express your answer as a common fraction.

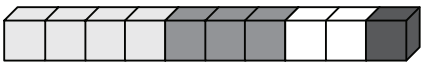


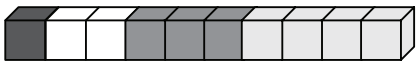
25. _____ The sum of five consecutive, positive even integers is a perfect square. What is the smallest possible integer that could be the least of these five integers?

26. _____ If $12_3 + 12_5 + 12_7 + 12_9 + 12_x = 101110_2$, what is the value of x, the base of the fifth term?

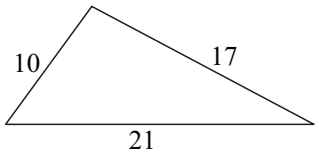
27. _____ A box contains r red balls and g green balls. When r more red balls are added to the box, the probability of drawing a red ball at random from the box increases by 25%. What was the probability of randomly drawing a red ball from the box originally? Express your answer as a common fraction.

28. _____ arrangements The game of *Connex* contains one 4-unit piece, two identical 3-unit pieces, three identical 2-unit pieces and four identical 1-unit pieces. How many different arrangements of pieces will make a 10-unit segment? The 10-unit segments consisting of the pieces 4-3-2-1 and 1-2-3-4 are two such arrangements to include.





29. _____ units^2 In square units, what is the largest possible area a rectangle inscribed in the triangle shown here can have?



30. (,) A line segment with endpoints A(3, 1) and B(2, 4) is rotated about a point in the plane so that its endpoints are moved to A′(4, 2) and B′(7, 3), respectively. What are the coordinates of the center of rotation? Express your answer as an ordered pair.

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: Express 8 divided by 12 as a common fraction. *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: Express 12 divided by 8 as a common fraction. *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter of $\frac{1}{4}$ as a common fraction in terms of π. *Answer:* $\frac{1+2\pi}{8}$

Problem: Express 20 divided by 12 as a mixed number. *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:
Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: Evaluate $\sqrt{15}\times\sqrt{5}$. *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they may both be omitted. Examples:
Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:
Problem: Write 6895 in scientific notation. *Answer:* 6.895×10^3
Problem: Write 40,000 in scientific notation. *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.