MATHCOUNTS®

2014
■ Chapter Competition ■
Sprint Round
Problems 1−30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

Raytheon

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02-C14SPR

1. _____ What is the value of ❀ if $\dfrac{❀+2}{3} = 5$?

2. $ _____ Arturo bought a shirt on sale for 30% off its original price of $10. He then sold it for a price that was 50% more than the price he paid for it. How much did Arturo sell the shirt for?

3. _____ quarts Sally is mixing a cleaning solution. She wants the ratio of bleach to water to be 1:8. How many quarts of bleach should she add to 24 quarts of water?

4. _____ triangles A square is divided by its four lines of symmetry. Triangles of different sizes are formed. How many total triangles are formed?

5. _____ inches There are already 4.5 inches of snow on the ground. Snow is falling at an average rate of 1.5 inches per hour and is predicted to continue for nine more hours. If the prediction is correct, how many inches of snow will be on the ground when the snow stops after 9 hours?

6. _____ If x and y are positive integers such that $xy = 100$, what is the positive difference between the maximum and minimum possible values of $x + y$?
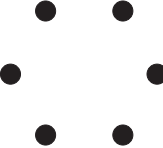
7. _____ bugs   If a bat catches a total of 560 bugs in 7 nights, and each night the bat catches the same number of bugs, how many bugs did the bat catch on the fifth night?

8. _____ cm   Kaylee cut a 20-cm by 20-cm piece of paper along a diagonal to form two congruent triangles. What is the length of the hypotenuse of one of the triangles? Express your answer in simplest radical form.

9. _____   If $a \odot b = \dfrac{ab}{a+b}$ and $a \odot 4 = 3$, what is the value of a?
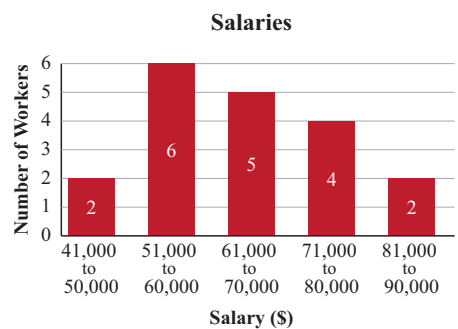
10. _____   What is the probability Kai will select a red or blue marble when he selects one marble from a jar containing three green, two red and five blue marbles? Express your answer as a common fraction.

11. _____ lines   There are six points in a plane, and no three of these points are collinear. How many distinct lines are determined by all possible pairs of these six points?

12. $ _____   The bar graph shows the salaries of workers at the Oro De Montana office, and all the salaries are multiples of $1000. What is the absolute value of the difference between the greatest possible mean salary for the employees and the least possible mean salary?

Salaries

| Salary ($)	Number of Workers
41,000 to 50,000	2
51,000 to 60,000	6
61,000 to 70,000	5
71,000 to 80,000	4
81,000 to 90,000	2

13. _____ What is the sum of the first 2000 positive integers?

14. _____ years On the ceiling of a cavern, a stalactite grows downward 0.004 *inches* per year. Ten *feet* directly below on the cavern's floor, a stalagmite grows upward 0.006 *inches* per year. How many years will it take them to meet?

15. _____ The product of three distinct positive integers is 144. If the sum of the three integers is 26, what is the sum of their squares?

16. _____ ordered triples How many ordered triples (x, y, z) of positive integers have the property that $x + y + z = 6$?

17. _____ If the slope of a line is k, and the line passes through the points $(0, k + 3)$ and $(-3, 0)$, what is the value of k? Express your answer as a common fraction.

18. _____ permu-tations How many permutations of the digits 1, 2, 3, 4 have 1 before 3 and 2 before 4? One such permutation to include is 2134.

19. _____ What is the value of $\left(\sqrt[3]{-8} \right)^4$?

20. (__ , __) The line $y = -\dfrac{1}{3}x + 4$ intersects a perpendicular line at (6, 2). What is the y-intercept of that perpendicular line? Express your answer as an ordered pair.

21. _____ The nine regions on a spinner are numbered with positive integers 1 through 9, and the probability of the spinner landing on a given region is proportional to the label for that region. For example, since $4 \times 2 = 8$, the probability of landing on 8 is twice the probability of landing on 4. What is the probability that this spinner will land on an odd number? Express your answer as a common fraction.

22. _____ The length of the diagonal of rectangle ABCD equals half the rectangle's perimeter less four-fifths of the length of the shorter side. What is the ratio of the length of the shortest side of the rectangle to the length of its longest side? Express your answer as a common fraction.

23. _____ units An equilateral triangle, with sides of length 2 units has a circular arc centered at B, inscribed as shown. What is the length of a segment drawn parallel to side AC with endpoints D and E? Express your answer in simplest radical form.
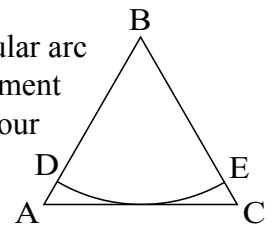
24. _____ units On a number line, how many units apart are the two points that are twice as far from 0 as they are from 9?

25. _____ For what positive integer n does $3n^3 + 3n^2 + 4n = n^n$?

26. _____ combi- How many different combinations of three numbers can be selected from the set
     nations $\{1, 2, 3, 4, 5, 6, 7, 8\}$ so that the numbers could represent the side lengths of a
     triangle?

27. _____ If $\dfrac{1}{x} + \dfrac{3}{y} = \dfrac{3}{4}$ and $\dfrac{3}{x} - \dfrac{2}{y} = \dfrac{5}{12}$, what is the value of $x + y$?

28. _____ What is the smallest prime number that divides some number of the form
     $424242\ldots42 + 1$ or of the form $424242\ldots42 - 1$?

29. _____ units Point E lies within rectangle ABCD. If AE = 7, BE = 5 and CE = 8, what is DE?
     Express your answer in simplest radical form.

30. _____ What is the value of $\sqrt{1 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}}$, where all subsequent numbers in
     the expression are 2s? Express your answer in simplest radical form.

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: Express 8 divided by 12 as a common fraction.   *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: Express 12 divided by 8 as a common fraction.   *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter

of $\frac{1}{4}$ as a common fraction in terms of π.   *Answer:* $\frac{1+2\pi}{8}$

Problem: Express 20 divided by 12 as a mixed number.   *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:
Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: Evaluate $\sqrt{15}\times\sqrt{5}$.   *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* are both zero, in which case they may both be omitted. Examples:
Acceptable: 2.35, 0.38, .38, 5.00, 5   *Unacceptable:* 4.9, 8.0

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:
Problem: Write 6895 in scientific notation.   *Answer:* 6.895×10^3
Problem: Write 40,000 in scientific notation.   *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.