
MATHCOUNTS®

2014
■ State Competition ■
Sprint Round
Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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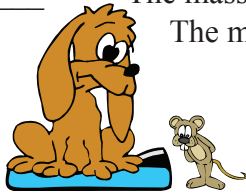
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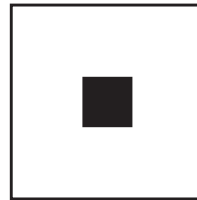
03-S14SPR

1. _____ The mass of a mouse is 25 grams, and the mass of a small dog is 5000 grams.
The mass of the dog is how many times the mass of the mouse?

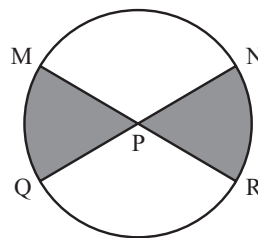


2. _____ What is the product of all the solutions to $|x - 3| = 4$?

3. _____ A square target has side length 16 inches. In the center of the target is a smaller square, as shown, with side length 4 inches. If a dart randomly hits within the larger square, what is the probability that it will land in the smaller square?
Express your answer as a common fraction.



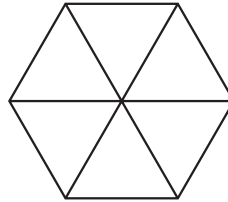
4. _____ Circle P has diameter 2 cm and $m\angle MPN = 120^\circ$. If the shaded region has area $k\pi$ cm², what is the value of k ? Express your answer as a common fraction.



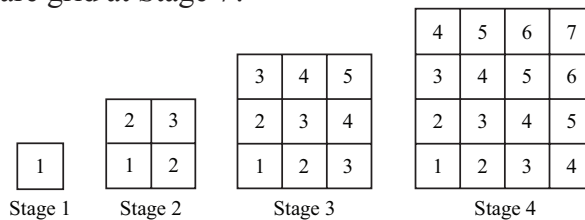
5. _____ If $f(x) = \frac{f(x+1)}{2}$ and $f(2) = 4$, then what is the value of $f(10) - f(7)$?

6. _____ If n represents the number of seconds in a day, what is the largest prime factor of n ?

7. _____ polygons How many convex polygons with an even number of sides are in the figure shown?



8. _____ Consider the pattern of square grids shown. The sum of the numbers in the square grid at Stage 3 is 27. If the pattern continues, what will be the sum of the numbers in the square grid at Stage 7?



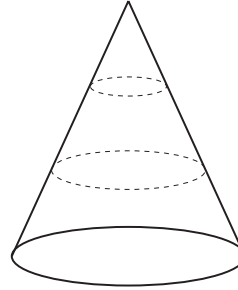
9. _____ terms Consider the sequence 1, 3, 4, 7, 11, 18, 29, ... in which each term is the sum of the two previous terms after the first two terms. How many of the first 100 terms of this sequence are multiples of 5?

10. _____ Three different gifts were bought to be distributed to three children. Each gift box was labeled with the recipient's name. Unfortunately, the gifts were later wrapped, covering the labels. If each child is given one of the three gifts at random to open, what is the probability that no child opens a gift labeled with his or her name? Express your answer as a common fraction.



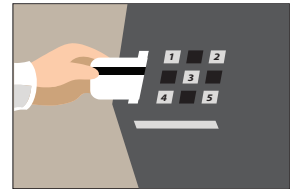
11. _____ in^2 The side lengths of three similar regular pentagons are in the ratio 2:5:7. The area of the smallest pentagon is 40 in^2 . What is the area of the largest pentagon?
12. _____ units^2 What is the area of the quadrilateral with vertices $(0, 0)$, $(1, 2)$, $(3, 4)$ and $(6, 5)$? Express your answer as a mixed number.
13. _____ The non-negative integers a, b, c, d and e form an arithmetic sequence. If their sum is 440, what is the largest possible value for e ?
14. _____ The six-digit integer "789,XYZ" consists of six *distinct* digits and is divisible by 7, 8 and 9. What is the three-digit integer "XYZ"?
15. _____ The nonzero roots of the equation $x^2 + 6x + k = 0$ are in the ratio 2:1. What is the value of k ?

16. _____ A cone is sliced by planes parallel to its base into three pieces of equal height. What fraction of the original volume is in the middle piece? Express your answer as a common fraction.



17. _____ A fair, six-sided die has two red, two blue and two yellow faces. The die is rolled three times. What is the probability of getting each color once? Express your answer as a common fraction.

18. _____ codes The First National Bank of Zyzylvaria requires each customer to choose a code comprising four distinct digits from 1 to 5, inclusive, where the positive difference between any two consecutive digits must be at least 2. How many such codes are possible?

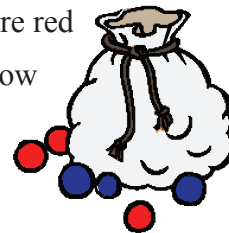


19. _____ students To the nearest whole percent, the composition of a particular class is 43% male students and 57% female students. What is the minimum number of students that could be in the class?

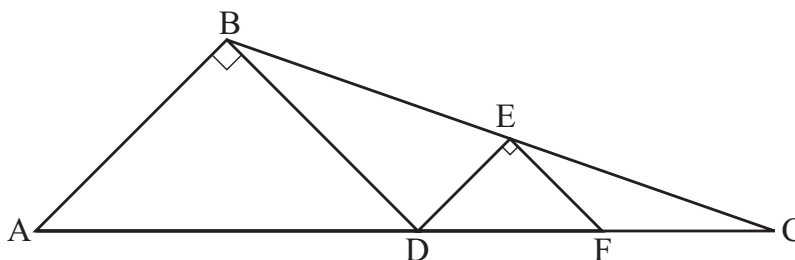
20. _____ base 5 In base 5, what is the value of $27_{10} \times 314_5$?

21. _____ If $x^2 + \frac{1}{x^2} = 3$ and $x > 0$, what is the value of $x + \frac{1}{x}$? Express your answer in simplest radical form.

22. _____ marbles A bag contains red marbles and blue marbles. If two marbles are chosen at random without replacement, the probability that both marbles are red is $\frac{1}{5}$, and the probability that both marbles are blue is also $\frac{1}{5}$. How many marbles are in the bag?



23. _____ units Triangles ABD and DEF are isosceles right triangles. Points A, D, F and C are collinear, and points B, E and C are collinear. If $AB = BD = 4$ and $ED = EF = 2$, what is the length of segment AC? Express your answer in simplest radical form.



24. _____ digits How many digits are in the integer representation of 2^{30} ?

25. _____ cm^2 The diagonals of a parallelogram have lengths 82 cm and 30 cm. One altitude is 18 cm long. What is the smallest possible area of the parallelogram?

26. _____ ways Seven different prizes are to be distributed among three contest winners such that each winner receives at least one prize and each of the prizes goes to one of the three winners. In how many different ways can the prizes be distributed among the three winners?

27. _____ The fourth degree polynomial equation $x^4 - 7x^3 + 4x^2 + 7x - 4 = 0$ has four real roots, a , b , c and d . What is the value of the sum $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$? Express your answer as a common fraction.

28. _____ What is the remainder when 11^{12} is divided by 13?

29. _____ integers Let $f(n) = \begin{cases} n^2 + 1 & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$. For how many integers n from 1 to 100, inclusive, does $f(f(\dots f(n))) = 1$ for some number of applications of f ?

30. _____ units In pentagon ABCDE, $\angle E$ and $\angle C$ are right angles and $m\angle D = 120^\circ$. If $AB = 12$, $AE = BC = 18$ and $ED = DC$, what is ED ? Express your answer in simplest radical form.

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A “common fraction” is to be considered a fraction in the form $\pm \frac{a}{b}$, where a and b are natural numbers and $\text{GCF}(a, b) = 1$. In some cases the term “common fraction” is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form $\pm N \frac{a}{b}$, where N , a and b are natural numbers, $a < b$ and $\text{GCF}(a, b) = 1$. Examples:

Problem: Express 8 divided by 12 as a common fraction. *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$
Problem: Express 12 divided by 8 as a common fraction. *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$
Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter of $\frac{1}{4}$ as a common fraction in terms of π . *Answer:* $\frac{1+2\pi}{8}$
Problem: Express 20 divided by 12 as a mixed number. *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{1}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: Evaluate $\sqrt{15} \times \sqrt{5}$. *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$) $a.bc$, where a is an integer and b and c are digits.

The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they may both be omitted. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

Do not make approximations for numbers (e.g., π , $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the “rounding” a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: Write 6895 in scientific notation. *Answer:* 6.895×10^3

Problem: Write 40,000 in scientific notation. *Answer:* 4×10^4 or 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.

Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.