
MATHCOUNTS®

2015
■ State Competition ■
Sprint Round
Problems 1–30

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HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. _____ weeks Jared needs \$240 to buy a new bicycle. He has saved \$60 so far and saves \$9 per week. In how many weeks will Jared have saved enough to buy the bicycle?

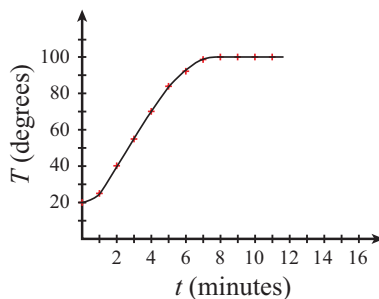


2. _____ Given that $2 = \frac{a}{20} = \frac{60}{b}$, what is the value of $a + b$?

3. _____ Given the function $f(x) = 2x^2 - 3x + 7$ with domain $\{-2, -1, 3, 4\}$, what is the largest integer in the range of f ?

4. _____ The first two terms of a sequence are 10 and 20. If each term after the second term is the average of all of the preceding terms, what is the 2015th term?

Water Temperature



5. _____ degrees

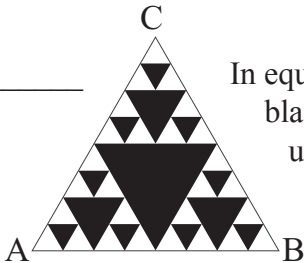
This graph shows the water temperature T degrees at time t minutes for a pot of water placed on a stove and heated to 100 degrees. On average, how many degrees did the temperature increase each minute during the first 8 minutes?

6. _____ In the table shown, $y = \frac{3x-1}{2}$. What is the value of t ?

x	y
1	1
s	4
t	$t + 4$

7. _____ downloads A streaming video service offers its customers two plans. The first plan has a \$24 registration fee, a fixed monthly rate of \$5 and a charge of \$0.75 for each video download. The second plan has a \$15 registration fee, a fixed monthly rate of \$3 and a charge of \$1.50 for each video download. For how many downloads would the two plans cost the same for a year of service?

8. _____ feet The length of a rectangle is three times its width. A new rectangle is created by decreasing the length of the original rectangle by 9 feet and increasing its width by 4 feet. The area of the new rectangle is the same as the area of the original rectangle. What is the perimeter of the new rectangle?

9. _____  In equilateral triangle ABC, shown here, each downward pointing black triangle has its vertices at the midpoints of the sides of a larger upward pointing white triangle. What fraction of the area of $\triangle ABC$ is white? Express your answer as a common fraction.

10. _____ units What is the greatest possible perimeter of an isosceles triangle with sides of length $5x + 20$, $3x + 76$ and $x + 196$?

11. _____ Consider an arithmetic sequence with $a_3 = 165$ and $a_{12} = 615$. For what value of n is $a_n = 2015$?

12. _____ If $\sqrt{x+7} = 2 + \sqrt{x}$, what is the value of x ? Express your answer as a common fraction.

13. _____ The line perpendicular to $2x - 2y = 2$, and with the same y -intercept, is graphed on the coordinate plane. What is the sum of its x - and y -intercepts?

14. _____ What is the units digit of the sum of the squares of the integers from 1 to 2015, inclusive?

15. _____ ways The doctor gave Amber ten vitamins, with instructions to take one or two each day until she runs out of vitamins. For example, Amber could take a vitamin a day for ten days, or she could take two the first day and one a day for the next eight days. A third way is to take one vitamin a day for eight days and two on the ninth day. Including the three examples given, in how many different ways can Amber take the ten vitamins?



16. _____ units What is the radius of a circle inscribed in a triangle with sides of length 5, 12 and 13 units?

17. _____ gumballs A bag initially had blue, red and purple gumballs in the ratio of 2:3:4. Five red gumballs are added to the bag. The probability of randomly drawing a red gumball is now 40%. How many gumballs are now in the bag?

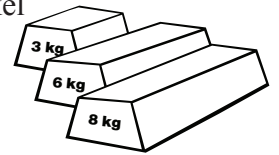
18. _____ The quadratic equation $x^2 + px + 2p = 0$ has solutions $x = a$ and $x = b$. If the quadratic equation $x^2 + cx + d = 0$ has solutions $x = a + 2$ and $x = b + 2$, what is the value of d ?

19. _____ cm² A rectangle of perimeter 22 cm is inscribed in a circle of area 16π cm². What is the area of the rectangle? Express your answer as a decimal to the nearest tenth.

20. _____ The units and tens digits of one two-digit integer are the tens and units digits of another two-digit integer, respectively. If the product of the two integers is 4930, what is their sum?

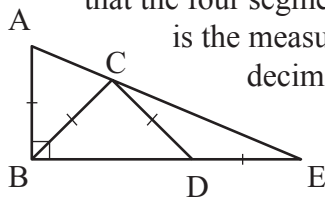
21. _____ Octavius has eight identical blue socks, six identical red socks, four identical black socks and two identical orange socks in his drawer. If he randomly selects two socks from his drawer, what is the probability that they will be the same color? Express your answer as a common fraction.

22. _____ small bars Iniki has large, medium and small metal bars. The large bars each weigh 8 kg. The medium bars each weigh 6 kg. The small bars each weigh 3 kg. Iron, nickel and lead are present in the ratio 4:1:3 in each large bar, 2:1:3 in each medium bar and 1:1:1 in each small bar. If Iniki wants to melt together a combination of bars to make an alloy that contains 40 kg of iron, 20 kg of nickel and 40 kg of lead, how many small bars will she have to use?



23. _____ If $x + \frac{1}{x} = 3$, what is the value of $x^4 + \frac{1}{x^4}$?

24. _____ degrees Points C and D are chosen on the sides of right triangle ABE, as shown, such that the four segments AB, BC, CD and DE each have length 1 inch. What is the measure of angle BAE, in degrees? Express your answer as a decimal to the nearest tenth.



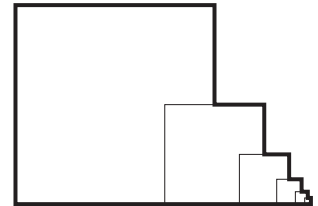
25. _____ people A restaurant sells three sizes of drinks: small for \$1.20, medium for \$1.30 and large for \$1.80. Each person at a table of ten ordered one drink, for a total cost of \$14.90, before sales tax. How many people ordered a large drink?

26. _____
arrange-
ments

Each of the 25 cells in a five-by-five grid of squares is filled with a 0, 1 or 2 in such a way that the numbers written in neighboring cells differ from the number in that cell by 1. Two cells are considered neighbors if they share a side. How many different arrangements are possible?

27. _____
units²

Starting with a square of side length 1, a square of side length $\frac{1}{2}$ is drawn so that it is bisected by a side of the original square, as shown. This process is repeated with a square of side length $\frac{1}{4}$ bisected by the square of side length $\frac{1}{2}$, and so on without end. What would be the area of such a figure, when generated as described? Express your answer as a common fraction.



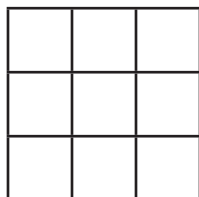
28. _____

In regular pentagon ABCDE, point M is the midpoint of side AE, and segments AC and BM intersect at point Z. If $ZA = 3$, what is the value of AB? Express your answer in simplest radical form.

29. _____

Fifty tickets numbered with consecutive integers are in a jar. Two are drawn at random and without replacement. What is the probability that the absolute difference between the two numbers is 10 or less? Express your answer as a common fraction.

30. _____
ways



How many ways are there to arrange the digits 1 through 9 in this 3×3 grid, such that the numbers are increasing from left to right in each row and increasing from top to bottom in each column?

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A “common fraction” is to be considered a fraction in the form $\pm \frac{a}{b}$, where a and b are natural numbers and $\text{GCF}(a, b) = 1$. In some cases the term “common fraction” is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N , a and b are natural numbers, $a < b$ and $\text{GCF}(a, b) = 1$. Examples:

Problem: Express 8 divided by 12 as a common fraction. *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$
Problem: Express 12 divided by 8 as a common fraction. *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$
Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter of $\frac{1}{4}$ as a common fraction in terms of π . *Answer:* $\frac{1+2\pi}{8}$
Problem: Express 20 divided by 12 as a mixed number. *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{1}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: Evaluate $\sqrt{15}\times\sqrt{5}$. *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$) $a.bc$, where a is an integer and b and c are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they may both be omitted. Answers in the form (\$) $a.bc$ should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π , $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the “rounding” a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: Write 6895 in scientific notation. *Answer:* 6.895×10^3

Problem: Write 40,000 in scientific notation. *Answer:* 4×10^4 or 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.

Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.