Divisibility

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1 Divisibility rules

To determine if a number is divisible by another number, there are a set of rules that dictate that. The ones mentioned in this page of notes are only one set of rules; there are many different rules invoving divisibility.

1.1 Divisibility by 2

Any number that ends in an even integer: $2, 4, 6, 8, 0$ will be divisible by 2.

1.2 Divisibility by 3

For a number to be divisible by 3, the sum of the digits must be a multiple of 3. ie. 15678 is divisible by 3 because $1 + 5 + 6 + 7 + 8 = 27$ and 27 is a multiple of 3.

1.3 Divisibility by 4

For a number ot be divisible by 4, it must be an even number and the last two digits together must be a multiple of 4. ie. 15383744 is divisible by 4 becuase 44, the last two digits is a multiple of 4.

1.4 Divisibility by 5

For a number to be divisible by 5, it must end in a 5 or 0.

1.5 Divisibility by 6

For a number to be divisible by 6, it must be both divisible by 2 and 3.

1.6 Divisibility by 7

Divisibility by 7 is one of the hardest to determine. There are multiple ways to determine, none of which are easy. One way is to take blocks of 3 numbers from the left and alternately sum them. If the alternating sum is a multiple of 7,

then is it divisible by 7. ie. 1,369,851 is divisible by 7 because $851 - 369 + 1 = 483 = 7 * 69$
Another way to determine divisibility is to subtract 2 times the last digit from the rest of the number. If the resulting answer is divisible by 7, then the number is divisible by 7. ie. 483 is divisible by 7 because $48 - 2 * 3 = 42$ and 42 is a multiple of 7.

1.7 Divisibility by 8

The divisibility rule for 8 is similar to the divisibility rule for 4. For a number to be divisible by 8, the number formed by the last three digits must be divisible by 8. If the number is not easily identifiable as a multiple by 8, you can add the last digit to two times the rest of the number. ie.341,176 is divisible by 8 because looking at the last three digits, 176, we see that is divisible by 8 because $17 * 2 + 6 = 40$ which is an obvious multiple of 8.

1.8 Divisibility by 9

The divisibility rule for 9 is a very simple one; it is very very similar to the divisibility rule by 3. If the sum of the digits is divisible by 9, then the number is divisible by 9. ie. 97349571 is divisible by 9 becuase 9+7+3+4+9+5+7+1 = 45 which is a multiple of 9.

1.9 Divisibility by 11

The divisibility rule for 11 is a very unique and interesting one. For a number to be divisible by 9, the difference between the "even" and "odd" digits must be 0 or a multiple of 9. ie. 341 is divisible by 11 because $(3 + 1) - (4) = 0$. Let's take a larger number: 3,452,471. Is it divisible by 11? Yes, it actually is because $(3 + 5 + 4 + 1) - (4 + 2 + 7) = 0$.