Exponents, Radicals, and Logarithms

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(Solutions to Practice Problems are at the end)

1 Exponents

Definition 1 (Exponent) *An exponent is a function expressed as a^b where a and b can be either any real number, constant, or variable.*

1.1 Laws of Exponents

Exponents are a special set of functions in mathematics. There are laws that dictate how exponential functions interact with each other.

1. $a^1 = a$
2. $a^b * a^b = a^{2b}$
3. $\frac{a^b}{a^c} = a^{b-c}$
4. $a^{b^c} = a^{(b^c)}$
5. $(\frac{a}{b})^c = \frac{a^c}{b^c}$
6. $a^0 = 1, a \neq 0$
7. $a^{-1} = \frac{1}{a}$
8. $x^{\frac{a}{b}} = \sqrt[b]{x^a}$
9. $0^a = 0, a \neq 0$
10. 0^0 is indeterminate; it is not defined by a single value.
11. $(ab)^c = a^c b^c$

1.2 Practice Problems

Simplify

1. 2^{3^2}
2. 5^{0^3}
3. $\frac{3^8 4^{10}}{3^6 4^9}$
4. $4^4 8^2 \frac{1}{2^{14}}$
5. $\frac{x^y y^z z^x}{x^{y-1} y^{z-1} z^{x-1}}$

2 Logarithms

Definition 2 (Logarithms) *Logarithms are a special type of function expressed as* $\log_a b$. *There is a set of laws that dictate how exponential functions interact with each other and with exponential functions.*

2.1 Laws of Logarithms

1. $\log_a b = c \Rightarrow a^c = b$
2. $\log_a b + \log_a c = log_a bc$
3. $\log_a b - \log_a c = log_a(\frac{b}{c})$
4. $\log_a b^c = c \log_a b$
5. $a^{\log_a b} = b$
6. $\ln a = \log_e a \Rightarrow \ln e = 1$
7. $\log_a 1 = 0$

2.2 Practice Problems

Simplify as much as possible

1. $\log_3 x + \log_3 y$
2. $\log_4 xy - \log_4 yz$
3. $\log_5 16 - \log_5 4$
4. $2^{\log_2 xyz}$
5. $\frac{\ln e^3}{\ln e^6}$

3 Radicals

Definition 3 (Radicals) *Radicals are a special type of function expressed as* $\sqrt[n]{x}$*. There is a set of laws that dictate how radical functions interact with each other and with exponential functions.*

3.1 Laws of Radicals

1. $\sqrt[n]{x} * \sqrt[n]{y} = \sqrt[n]{xy}$
2. $\sqrt[n]{x^m} = x^{\frac{m}{n}}$
3. $\sqrt[n]{x^n} = x$
4. $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$
5. $\sqrt[n]{x} * \sqrt[n]{x} = \sqrt[mn]{x^{m+n}}$
6. $\frac{\sqrt[n]{x}}{\sqrt[m]{x}} = \sqrt[mn]{x^{m-n}}$

3.2 Practice Problems

Simplify

1. $\sqrt[3]{5} * \sqrt[3]{7}$
2. $\sqrt[2]{4^4}$
3. $\frac{\sqrt{4}}{\sqrt{9}}$
4. $\sqrt[7]{2} * \sqrt[3]{2}$
5. $\frac{\sqrt[3]{5}}{\sqrt[2]{5}}$

4 Solutions

4.1 1.2

1. $2^9 \Rightarrow 512$
2. 1
3. $3^2 4 \Rightarrow 36$
4. 1
5. xyz

4.2 2.2

1. $\log_3 xy$
2. $\log_4 \frac{xy}{yz} \Rightarrow \log_4 \frac{x}{z}$
3. $log_5 4$
4. xyz
5. $\frac{1}{2}$

4.3 3.2

1. $\sqrt[3]{35}$
2. 8
3. $\frac{2}{3}$
4. $\sqrt[21]{2^{10}} \Rightarrow \sqrt[21]{1024}$
5. $\sqrt[6]{5}$